Hybrid Beamforming Design for C-RAN Based mmWave Cell-Free Systems

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Abstract-This paper considers the cloud radio access network (C-RAN) based millimeter-wave (mmWave) cell-free communications, where multiple remote radio heads (RRHs) are distributed to provide reliable communication links to users via analog beamforming and connected to centralized baseband unit (BBU) which carries out digital signal processing. We aim to jointly design the user association and analog/digital hybrid beamforming along with fronthaul compression to maximize the minimum signal to interference-plus-noise ratio (SINR) among users while satisfying the fronthaul capacity constraints. To solve this difficult combinatory problem, we propose to first obtain the user association and analog beamforing to maximize the minimum beamforming gain among users. Then, given the effective baseband channel, the digital beamformer and quantization noise covariance matrix still cannot be calculated directly due to the non-convexities of objective function and fronthaul constraint. To efficiently solve this problem, we transform the objective function into convex terms based on fractional programming method and iteratively calculate the digital beamformer and quantization noise covariance matrix until convergence is achieved. Simulation results show that the proposed algorithm can achieve comparable performance to the full-digital beamforming.

Index Terms—Millimeter-wave communications, hybrid beamforming, cell-free network, cloud radio access network (C-RAN).

I. INTRODUCTION

The need for higher data rates has always been the key promoter for wireless network evolution. The demand for wireless capacity will continue to grow, which motivates exploring higher frequency bands beyond sub-6 GHz, e.g. millimeter wave (mmWave) frequencies, to exploit the abundant available spectrum [1]. However, the mmWave signals experience severe path loss and penetration due to the short wavelength, which frustrates the real-world deployment of mmWave communication systems. To deal with these problems and make mmWave communication a reality in 5G and beyond systems, multiple-input multiple-output (MIMO) [2],

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[3] and network densification [4], [5] are key technologies to combat the significant attenuation of mmWave channels and sustain the spectral efficiency.

To fully take the advantages of multiple antennas while maintain affordable hardware cost, hybrid beamforming architecture is employed in mmWave communications which realizes analog beamforming through a large scale phase shifter network and digital beamforming using a small number of radio frequency (RF) chains. Nevertheless, because the mmWave beams are narrow and have weak penetrability, the communication links to users at cell-edge may be blocked by other obstacles and these users would suffer from poor communication qualities. In addition, with dense deployment of mmWave base stations, the inter-cell interference also increases which becomes the main impediment to further enhance the system throughput. In order to deal with these difficulties and guarantee the quality of service of users, cellfree structure, which breaks the restrictions of traditional cellular bounds, has been regarded as a promising architecture for future mmWave networks [6], [7]. Cloud radio access network (C-RAN) provides as an efficient platform for implementing cooperations [8], in which multiple remote radio heads (RRHs) are distributed to cooperatively perform radio frequency (RF) operations under the control of centralized baseband unit (BBU). Practically, the BBU is connected with each RRH via a capacity-limited fronthaul link and the baseband signals are usually quantized and compressed prior to being delivered to the RRH.

Hybrid beamforming for C-RAN based cell-free systems is different from conventional cellular systems since the fronthaul constraints and user association also need to be taken into account in the beamforming design. Precoder and fronthaul compression design for C-RAN systems have been investigated in [9], where the beamformer and quantization noise covariance matrix are iteratively calculated based on successive convex approximation (SCA) and weighted minimummean-square-error (WMMSE) methods to maximize the sumrate. The extension of this work to hybrid beamforming is investigated in [10], where the analog and digital beamformer,



Fig. 1. Hybrid beamforming for C-RAN based mmWave cell-free systems.

as well as fronthaul compression are separately calculated based on WMMSE approach. In [11], the authors propose a two-stage approach to maximize the sum-rate, in which the analog beamformer is first determined based on a codebook, followed by digital beamforming and compression design based on semidefinite relaxation and convex approximation. Power minimization problem is considered in [12], and the authors proposed to transform the non-convex problem into a solvable second-order cone (SOC) fashion and jointly accomplish beamforming design and active RRH selection to minimize the energy consumption.

While the sum-rate maximization problem for C-RAN based mmWave cell-free communication has been extensively studied, the max-min fairness metric has not been considered yet. In this paper, we investigate the joint design of user association and hybrid beamforming, along with fronthaul compression for C-RAN based mmWave cell-free systems to maximize the minimum signal to interference-plus-noise ratio (SINR) among users. To the best of our knowledge, the max-min fair design for C-RAN communication systems has not been studied before. The objective function is a combinatory problem whose optimal solution cannot be obtained directly. We thus propose to separately design the analog and digital signal processing. In particular, a low-complexity user association and analog beamforming design algorithm is first introduced to maximize the minimum beamforming gain. Then, with effective baseband channel, the designs of digital beamforming and fronthaul compression are still intractable due to the non-convexities of objective function and fronthaul constraint. We propose a fractional programming based algorithm to transform the non-convex objective function into convex terms and calculate the digital beamfomring and quantization noise covariance matrix iteratively until convergence is achieved. Simulation results illustrate the effectiveness of the proposed algorithm.

II. SYSTEM MODEL

We consider a C-RAN based mmWave cell-free communication system with hybrid beamforming as shown in Fig. 1. The BBU transmits signals to K single-antenna users through M cooperative RRHs, using the same time-frequency resource. We assume the *i*-th RRH is connected to the BBU via a fronthaul link of capacity C_i b/s/Hz, i = 1, ..., M. Moreover, each RRH is equipped with $N_{\rm t}$ antennas and $N_{\rm RF}$ radio frequency (RF) chains, $N_{\rm RF} \leq K \leq M N_{\rm RF}$. In the downlink transmission, the received signal at the k-th user is given as

$$y_k = \sum_{i=1}^{M} \mathbf{h}_{i,k}^H \mathbf{x}_i + n_k, \tag{1}$$

where $\mathbf{x}_i \in \mathbb{C}^{N_t \times 1}$ denotes the transmitted signal of the *i*-th RRH subject to the transmit power constraint $\mathbb{E} ||\mathbf{x}_i||^2 \leq P_i$ and P_i is the maximum transmit power at the *i*-th RRH. $n_k \sim \mathcal{CN}(0, \sigma_k^2)$ is the Gaussian additive noise at the *k*-th user. $\mathbf{h}_{i,k}$ represents the mmWave channel vector between the *i*-th RRH and the *k*-th user, which can be characterized by the classic geometric channel model with *L* paths channel model [13]

$$\mathbf{h}_{i,k} = \sqrt{\frac{N_{\mathrm{t}}}{L}} \sum_{l=1}^{L} \alpha_l \mathbf{a}(\theta_l).$$
(2)

In (2), $\alpha_l \sim C\mathcal{N}(0, 1)$ and θ_l are the complex gain and angle of departure (AoD) of the *l*-th propagation path, respectively. The array response vector $\mathbf{a}(\theta)$ depends on the antenna array geometry. We assume that the commonly used uniform linear array (ULA) is employed, and $\mathbf{a}(\theta)$ can be written as

$$\mathbf{a}(\theta) = \frac{1}{\sqrt{N_{\rm t}}} [1, e^{j\frac{2\pi}{\lambda}d\sin(\theta)}, \dots, e^{j(N_{\rm t}-1)\frac{2\pi}{\lambda}d\sin(\theta)}]^T, \quad (3)$$

in which λ is the signal wavelength, and d is the distance between antenna elements.

Based on the C-RAN architecture, the transmitted signals \mathbf{x}_i at the *i*-th RRH is the processing result of hybrid beamforming, where the digital signal processing is performed at the BBU and each RRH carries out analog beamforming. To be specific, the BBU first precodes the symbol s_k of each user, k = 1, ..., K, which can be written as

$$\mathbf{x}_{D,i} = \sum_{k \in \mathcal{K}_i} \mathbf{f}_{\mathrm{BB}_{i,k}} s_k,\tag{4}$$

where $\mathbf{f}_{\text{BB}_{i,k}} \in \mathbb{C}^{N_{\text{RF}} \times 1}$ is the digital beamforming vector for user-k via the *i*-th RRH. \mathcal{K}_i indicates the set of users served by RRH-*i*. We assume the number of users served by each RRH is equal to the number of RF chains, i.e., $|\mathcal{K}_i| \leq N_{\text{RF}}$. Since the BBU delivers the signals to each RRH via a fronthaul link with limited capacity C_i , $\mathbf{x}_{D,i}$ has to be quantized and compressed prior to being transferred to each RRH:

$$\widetilde{\mathbf{x}}_{D,i} = \mathbf{x}_{D,i} + \mathbf{q}_i,\tag{5}$$

where $\mathbf{q}_i \sim \mathcal{CN}(\mathbf{0}, \mathbf{\Omega}_i)$ is the quantization noise vector. The data transmission rate after compression should satisfy the fronthaul link capacity constraint to guarantee a reliable information recovery at each RRH, which is given by

$$c_{i}(\mathbf{F}_{\mathrm{BB}_{i}}, \mathbf{\Omega}_{i}) = \log_{2} \frac{\left| \sum_{k \in \mathcal{K}_{i}} \mathbf{f}_{\mathrm{BB}_{i,k}} \mathbf{f}_{\mathrm{BB}_{i,k}}^{H} + \mathbf{\Omega}_{i} \right|}{|\mathbf{\Omega}_{i}|} \leq C_{i}, \forall i. (6)$$

At each RRH, the received baseband signal is further precoded by analog beamformer $\mathbf{F}_{\mathrm{RF}_i} \triangleq [\mathbf{f}_{\mathrm{RF}_{i,1}} \dots \mathbf{f}_{\mathrm{RF}_{i,N_{\mathrm{RF}}}}] \in \mathbb{C}^{N_{\mathrm{t}} \times N_{\mathrm{RF}}}$, and the signal to be transmitted from the *i*-th RRH is given as

$$\mathbf{x}_{i} = \mathbf{F}_{\mathrm{RF}_{i}} \mathbf{x}_{D,i}$$
$$= \mathbf{F}_{\mathrm{RF}_{i}} \mathbf{x}_{D,i} + \mathbf{F}_{\mathrm{RF}_{i}} \mathbf{q}_{i}.$$
 (7)

Therefore, by plugging the above expression into (1), the received signal at the k-th user can be rewritten as

$$y_{k} = \sum_{i=1}^{M} \mathbf{h}_{i,k}^{H} \mathbf{x}_{i} + n_{k}$$
$$= \sum_{i=1}^{M} \mathbf{h}_{i,k}^{H} \mathbf{F}_{\mathrm{RF}_{i}} \mathbf{F}_{\mathrm{BB}_{i}} \mathbf{s}_{i} + \mathbf{h}_{i,k}^{H} \mathbf{F}_{\mathrm{RF}_{i}} \mathbf{q}_{i} + n_{k}, \qquad (8)$$

where $\mathbf{F}_{BB_i} \triangleq [\mathbf{f}_{BB_{i,1}}, \dots, \mathbf{f}_{BB_{i,N_{RF}}}]$. The total power consumed by each RRH is expressed as

$$p_{i}(\mathbf{F}_{\mathrm{RF}_{i}}, \mathbf{F}_{\mathrm{BB}_{i}}, \mathbf{\Omega}_{i}) = \mathbb{E} \|\mathbf{x}_{i}\|^{2}$$

= trace $(\mathbf{F}_{\mathrm{RF}_{i}} \mathbf{F}_{\mathrm{BB}_{i}} \mathbf{F}_{\mathrm{BB}_{i}}^{H} \mathbf{F}_{\mathrm{RF}_{i}}^{H}) + \operatorname{trace} (\mathbf{F}_{\mathrm{RF}_{i}} \mathbf{\Omega}_{i} \mathbf{F}_{\mathrm{RF}_{i}}^{H}).$ (9)

Given the received signal at each user in (8), the SINR of the k-the user γ_k can be calculated as

$$\gamma_{k} = \frac{\left|\sum_{i=1}^{M} u_{i,k} \mathbf{h}_{i,k}^{H} \mathbf{F}_{\mathrm{RF}_{i}} \mathbf{f}_{\mathrm{BB}_{i,k}}\right|^{2}}{\sum_{l \neq k} \left|\sum_{i=1}^{M} u_{i,l} \mathbf{h}_{i,k}^{H} \mathbf{F}_{\mathrm{RF}_{i}} \mathbf{f}_{\mathrm{BB}_{i,l}}\right|^{2} + \sum_{i=1}^{M} \mathbf{h}_{i,k}^{H} \mathbf{F}_{\mathrm{RF}_{i}} \mathbf{\Omega}_{i} \mathbf{F}_{\mathrm{RF}_{i}}^{H} \mathbf{h}_{i,k} + \sigma_{k}^{2}}$$
(10)

where $u_{i,k}$ is the user association indicator and

$$u_{i,k} = \begin{cases} 1, & \text{if } k \in \mathcal{K}_i, \\ 0, & \text{if } k \notin \mathcal{K}_i. \end{cases}$$
(11)

We aim to jointly design user association indicator $\{u_{i,k}\}_{i,k}$, hybrid beamforming $\{\mathbf{F}_{\mathrm{RF}_i}\mathbf{F}_{\mathrm{BB}_i}\}_{i=1}^M$, and quantization noise covariance matrix $\{\mathbf{\Omega}_i\}_{i=1}^M$ to maximize the minimum SINR among users. The objective function can be formulated as

$$\max_{\{u_{i,k}\}_{i,k}, \{\mathbf{F}_{\mathrm{RF}_i}, \mathbf{F}_{\mathrm{BB}_i}, \mathbf{\Omega}_i\}_{i=1}^M} \min_{k} \gamma_k$$
(12a)

s.t.
$$u_{i,k} \in \{0,1\}, \ \forall i,k,$$
 (12b)

$$\sum_{k=1}^{K} u_{i,k} = N_{\rm RF}, \ \forall i, \tag{12c}$$

$$c_i(\mathbf{F}_{\mathrm{BB}_i}, \mathbf{\Omega}_i) \le C_i, \ \forall i,$$
 (12d)

$$p_i(\mathbf{F}_{\mathrm{RF}_i}, \mathbf{F}_{\mathrm{BB}_i}, \mathbf{\Omega}_i) \le P_i, \ \forall i,$$
 (12e)

$$|\mathbf{F}_{\mathrm{RF}_i}(m,n)| = \frac{1}{\sqrt{N_{\mathrm{t}}}}, \ \forall m,n,i.$$
(12f)

Note that the above problem is a combinatory design of four variables. In addition, the non-convex objective function and fronthaul constraint make obtaining the optimal solution even harder. In the next section, we aim to develop a sub-optimal solution with low-complexity but can achieve satisfactory performance.

III. MAX-MIN FAIR DESIGN FOR MMWAVE CELL-FREE Systems

In this section, we demonstrate the proposed algorithm which can be divided into two steps. First, we propose a low-complexity user association and analog beamforming design approach to maximize the minimum beamforming gain. Then, a fractional programming based scheme is developed to calculate the optimal digital beamforming and quantization noise covariance matrix.

A. Joint User Association and Analog Beamforming Design

We aim to jointly determine the user association indicator and analog beamformer to maximize the minimum accumulative beamforming gain among users, which can be formulated as

$$\max_{\{u_{i,k}, \mathbf{f}_{\mathrm{RF}_{i,k}}\}_{i,k}} \min_{k} \sum_{i=1}^{M} |u_{i,k} \mathbf{h}_{i,k}^{H} \mathbf{f}_{\mathrm{RF}_{i,k}}|^{2} \\
\text{s.t.} \quad (12b), (12c), \\
\mathbf{f}_{\mathrm{RF}_{i,k}} \in \mathcal{F}, \text{ if } u_{i,k} = 1, \quad (13)$$

where \mathcal{F} is the codebook for analog beamformer selection and is defined as [13]:

$$\mathcal{F} = \left\{ \mathbf{f}_n = \mathbf{a} \left(\frac{2\pi n}{N_{\text{res}}} \right), \ n = 1, \dots, N_{\text{res}} \right\}, \tag{14}$$

 $N_{\rm res}$ is the resolution to quantize the spatial angle. The above problem can be solved via brute-force searching over the combinations of beamformer selection from codebook and user association indicator. However, this will result in high computational complexity. In the following, we will introduce a low-complexity algorithm to efficiently solve (13).

The proposed scheme starts by ignoring the user association constraint, and setting all $u_{i,k} = 1$, $\forall i, k$. Under this condition, the optimal analog beamformer of each RRH-user pair is selected which results in the largest beamforming gain between the pair of RRH-*i* and user-*k*:

$$\tilde{\mathbf{f}}_{i,k} = \max_{\mathbf{f}_{i,k} \in \mathcal{F}} \left| \mathbf{h}_{i,k}^{H} \mathbf{f}_{i,k} \right|^{2}, \ \forall i,k.$$
(15)

We calculate the largest accumulative beamforming gain of each user ξ_k , k = 1, ..., K, which is given by

$$\xi_k = \sum_{i=1}^M \left| \mathbf{h}_{i,k}^H \tilde{\mathbf{f}}_{i,k} \right|^2.$$
(16)

Then, we reconsider the design of user association. We propose to successively remove the analog beamformers which generate least beamforming gain and set the corresponding user association indicator to zero. Particularly, given the optimal analog beamformers in (15), we find the RRH which contributes to the least gain to the k-th user:

$$\hat{i}_{k} = \arg\min_{i=1,\dots,M} \left| \mathbf{h}_{i,k}^{H} \tilde{\mathbf{f}}_{i,k} \right|^{2}, \ \forall k,$$
(17)

and calculate the corresponding remaining beamforming gains of each user when $\tilde{\mathbf{f}}_{i_k,k}$, $k = 1, \dots, K$, is eliminated:

$$\tilde{\xi}_k = \sum_{i \neq \hat{i}_k}^M \left| \mathbf{h}_{i,k}^H \tilde{\mathbf{f}}_{i,k} \right|^2, \ \forall k.$$
(18)

Note that if the constraint $\sum_{k=1}^{K} u_{\hat{i}_k,k} = N_{\rm RF}$ is already satisfied at the \hat{i}_k -th RRH, we then remove the beamformer which results in the second least beamforming gain whose RF chain constraint is not satisfied at the corresponding RRH. Next, we find the \bar{k} -th user whose remaining beamforming gain is the largest, and remove $\hat{f}_{\hat{i}_k,\bar{k}}$ from the \hat{i}_k -th RRH. $u_{\hat{i}_k,\bar{k}}$

is set to zero. Similarly, the beams are successively removed until the RF chain constraints of all the RRH are satisfied.

Finally, the user association indicators $\{u_{i,k}\}_{i,k}$ and the set of users served by RRH-*i* \mathcal{K}_i can be determined. The analog beamformer at the *i*-th RRH is $\mathbf{F}_{\mathrm{RF}_i}^{\star} = [\tilde{\mathbf{f}}_{i,\mathcal{K}_i(1)},\ldots,\tilde{\mathbf{f}}_{i,\mathcal{K}_i(N_{\mathrm{RF}})}]$. In addition, we can obtain the effective baseband channel between the *i*-th RRH and the *k*-th user as $\tilde{\mathbf{h}}_{i,k}^H \triangleq \mathbf{h}_{i,k}^H \mathbf{F}_{\mathrm{RF}_i}^{\star}$.

B. Digital Beamforming and Fronthaul Compression

With the effective baseband channel and user association indicators, in this subsection we discuss the digital beamforming and fronthaul compression design. For simplicity, the digital beamformer and effective baseband channel for the k-th user can be merged into a single vector as $\mathbf{f}_{\mathrm{BB}_k} \triangleq [\mathbf{f}_{\mathrm{BB}_{1,k}}^H, \dots, \mathbf{f}_{\mathrm{BB}_{M,k}}^H]^H$ and $\tilde{\mathbf{h}}_k \triangleq [\tilde{\mathbf{h}}_{1,k}^H, \dots, \tilde{\mathbf{h}}_{M,k}^H]^H$, respectively. In addition, we define the effective channel vector associated with the *l*-th user and *k*-th user as $\bar{\mathbf{h}}_{l,k} \triangleq$ $[u_{1,l}\tilde{\mathbf{h}}_{1,k}^H, \dots, u_{M,l}\tilde{\mathbf{h}}_{M,k}^H]^H$, $\forall l, k$, and the aggregated quantization noise covariance matrix $\boldsymbol{\Omega} \triangleq \operatorname{diag}(\boldsymbol{\Omega}_1, \dots, \boldsymbol{\Omega}_M)$. Thus, the digital beamforming and compression design problem can be rewritten as

$$\max_{\mathbf{F}_{\mathrm{BB}},\mathbf{\Omega}} \min_{k} \frac{\left| \bar{\mathbf{h}}_{k,k}^{H} \mathbf{f}_{\mathrm{BB}_{k}} \right|^{2}}{\sum_{l \neq k} \left| \bar{\mathbf{h}}_{l,k}^{H} \mathbf{f}_{\mathrm{BB}_{l}} \right|^{2} + \tilde{\mathbf{h}}_{k}^{H} \mathbf{\Omega} \tilde{\mathbf{h}}_{k} + \sigma_{k}^{2}}$$
s.t. $\log_{2} \left| \sum_{k \in \mathcal{K}_{i}} \mathbf{E}_{i}^{H} \mathbf{f}_{\mathrm{BB}_{k}} \mathbf{f}_{\mathrm{BB}_{k}}^{H} \mathbf{E}_{i} + \mathbf{\Omega}_{i} \right| - \log_{2} |\mathbf{\Omega}_{i}| \leq C_{i}, \forall i,$
 $p_{i}(\mathbf{F}_{\mathrm{RF}_{i}}, \mathbf{F}_{\mathrm{BB}_{i}}, \mathbf{\Omega}_{i}) \leq P_{i}, \quad \forall i,$
(19)

in which $\mathbf{F}_{\mathrm{BB}} \triangleq [\mathbf{f}_{\mathrm{BB}_1}, \dots, \mathbf{f}_{\mathrm{BB}_{N_{\mathrm{RF}}}}] \mathbf{E}_i \triangleq [\mathbf{0}_{N_{\mathrm{RF}} \times N_{\mathrm{RF}}(i-1)}^H \mathbf{I}_{N_{\mathrm{RF}}} \mathbf{0}_{N_{\mathrm{RF}} \times N_{\mathrm{RF}}(M-i)}^H]^H$. We can see that finding the global optimum solution of the above problem is challenging due to the non-convexities of the objective function and fronthaul link constraint. Inspired by the fractional programming algorithm introduced in [14], [15], we propose a low-complexity algorithm to find a stationary solution by transforming the objective function into a solvable fashion.

We first apply fractional programming [15] to transform the SINR expression into a convex term, which is expressed as

$$f_{k}^{w}(\mathbf{f}_{\mathrm{BB}_{k}}, \mathbf{\Omega}, \omega_{k}) = 2\mathfrak{Re}\left\{\omega_{k}\bar{\mathbf{h}}_{k,k}\mathbf{f}_{\mathrm{BB}_{k}}\right\} - |\omega_{k}|^{2}\sum_{l\neq k}^{K}\left|\bar{\mathbf{h}}_{l,k}\mathbf{f}_{\mathrm{BB}_{l}}\right| - |\omega_{k}|^{2}\tilde{\mathbf{h}}_{k}\mathbf{\Omega}\tilde{\mathbf{h}}_{k}^{H} - |\omega_{k}|^{2}\sigma_{k}^{2},$$
(20)

with ω_k introduced as an auxiliary variable to each user. We let $\boldsymbol{\omega} \triangleq [\omega_1, \dots, \omega_{N_{\mathrm{RF}}}]$. The original objective function is equivalent to iteratively calculating $\{\mathbf{F}_{\mathrm{BB}}, \Omega\}$ and $\boldsymbol{\omega}$ under the fronthaul link and power constraints. When the variables \mathbf{F}_{BB} and $\boldsymbol{\Omega}$ are fixed, given that ω_k is independent with each other, the optimal ω_k is obtained by maximizing f_k^{ω} over ω_k which is a convex problem. By setting $\frac{\partial f_k^{\omega}}{\omega_k} = 0$, the optimal value of ω_k is

$$\omega_{k}^{\star} = \frac{\mathbf{h}_{k,k}^{H} \mathbf{f}_{\mathrm{BB}_{k}}}{\sum_{l \neq k} \left| \bar{\mathbf{h}}_{l,k}^{H} \mathbf{f}_{\mathrm{BB}_{l}} \right|^{2} + \tilde{\mathbf{h}}_{k}^{H} \mathbf{\Omega} \tilde{\mathbf{h}}_{k} + \sigma_{k}^{2}}.$$
(21)

We then consider the optimization of $\{\mathbf{F}_{BB}, \Omega\}$ with fixed ω . Since the fronthaul link constraint is still non-convex with respect to \mathbf{F}_{BB} and Ω , we propose to further transform the fronthaul link constraint into a solvable function. For positive definite Hermitian matrices $\mathbf{A}, \mathbf{B} \in \mathbb{C}^{N \times N}$, we have

$$\log_2 |\mathbf{A}| \le \log_2 |\mathbf{B}| + \operatorname{trace} \{\mathbf{B}^{-1}\mathbf{A}\} - N$$
 (22)

with equality achieved when $\mathbf{A} = \mathbf{B}$. Therefore, based on the above relationship, we can approximate the fronthaul link constraint with

$$\log_2 \left| \sum_{k \in \mathcal{K}_i} \mathbf{E}_i^H \mathbf{f}_{\mathrm{BB}_k} \mathbf{f}_{\mathrm{BB}_k}^H \mathbf{E}_i + \mathbf{\Omega}_i \right| - \log_2 |\mathbf{\Omega}_i| \\ \leq g_i(\mathbf{f}_{\mathrm{BB}_k}, \mathbf{\Omega}_i, \mathbf{\Sigma}_i),$$
(23)

where $g_i(\mathbf{f}_{\mathrm{BB}_k}, \mathbf{\Omega}_i, \mathbf{\Sigma}_i)$ is the upper bound of fronthaul link capacity and is defined as

$$g_{i}(\mathbf{F}_{\mathrm{BB}}, \mathbf{\Omega}_{i}, \mathbf{\Sigma}_{i}) \triangleq \log_{2} |\mathbf{\Sigma}_{i}| - \log_{2} |\mathbf{\Omega}_{i}| - N_{\mathrm{RF}} + \operatorname{trace} \left\{ \mathbf{\Sigma}_{i}^{-1} \left(\sum_{k \in \mathcal{K}_{i}} \mathbf{E}_{i}^{H} \mathbf{f}_{\mathrm{BB}_{k}} \mathbf{f}_{\mathrm{BB}_{k}}^{H} \mathbf{E}_{i} + \mathbf{\Omega}_{i} \right) \right\}.$$
(24)

In (23), Σ_i is introduced as an auxiliary matrix and the equivalence holds only when

$$\boldsymbol{\Sigma}_{i}^{\star} = \sum_{k \in \mathcal{K}_{i}} \mathbf{E}_{i}^{H} \mathbf{f}_{\mathrm{BB}_{k}} \mathbf{f}_{\mathrm{BB}_{k}}^{H} \mathbf{E}_{i} + \boldsymbol{\Omega}_{i}.$$
 (25)

Finally, we formulate the problem

$$\max_{\mathbf{F}_{\mathrm{BB}}, \mathbf{\Omega}, \mathbf{\Sigma}} \min_{k} f_{k}^{w}$$

s.t. $g_{i}(\mathbf{F}_{\mathrm{BB}}, \mathbf{\Omega}_{i}, \mathbf{\Sigma}_{i}) \leq C_{i}, \forall i$
 $p_{i}(\mathbf{F}_{\mathrm{RF}_{i}}, \mathbf{F}_{\mathrm{BB}_{i}}, \mathbf{\Omega}_{i}) \leq P_{i}, \forall i.$ (26)

The above problem is convex with respect to any one of the variables when the other variables are fixed. Particularly, Σ_i is updated as in (25) with fixed { \mathbf{F}_{BB}, Ω }. On the other hand, { \mathbf{F}_{BB}, Ω } can be calculated based on the following convex problem:

$$\max_{\mathbf{F}_{\mathrm{BB}}, \mathbf{\Omega}} t$$
s.t. $f_k^w(\mathbf{f}_{\mathrm{BB}_k}, \mathbf{\Omega}, \omega_k) \ge t, \ \forall k,$

$$g_i(\mathbf{F}_{\mathrm{BB}}, \mathbf{\Omega}_i, \mathbf{\Sigma}_i) \le C_i, \ \forall i,$$

$$p_i(\mathbf{F}_{\mathrm{RF}_i}, \mathbf{F}_{\mathrm{BB}_i}, \mathbf{\Omega}_i) \le P_i, \ \forall i,$$

$$(27)$$

which can be solved via CVX. Therefore, the three variables can be iteratively optimized until convergency is achieved.



Fig. 2. Minimum SINR versus transmit power P with M = 3, $N_t = 16$, $N_{RF} = 4$, and K = 6.



Fig. 3. Minimum SINR versus the number of users K with P = 20 dBW, M = 3, $N_{\rm t} = 16$, and $N_{\rm RF} = 4$.

IV. SIMULATION RESULTS

In this section, we provide numerical results of the proposed schemes to evaluate the performance of the max-min fair design. We consider the system setting of K = 6 singleantenna users and M = 3 RRHs, each of which is equipped with $N_{\rm RF} = 4$ RF chains and $N_{\rm t} = 16$ antennas. For simplicity, we assume all RRHs have the same fronthaul link capacity and transmit power [10], i.e. $C_i = C$, $P_i = P$, $\forall i$. The noise power of each user is set as $\sigma_k = \sigma = 1$, $\forall k$.

Fig. 2 illustrates the minimum SINR versus transmit power P with C = 8,10 b/s/Hz. For comparison purpose, the performance of full digital beamforming is included as benchmark, in which we set $N_{\rm RF} = N_{\rm t} = 16$ and all users are simultaneously served by all RRHs. We can see that our proposed algorithm can achieve comparable performance to the full-digital case with much smaller number of RF chains, which demonstrates the effectiveness of the proposed user association and hybrid beamforming designs. In Fig. 3, we present the minimum SINR as a function of the number of users K with P = 20 dBW. It can be observed that the minimum SINR decreases with larger number of users

and our proposed algorithm can always achieve satisfactory performance.

V. CONCLUSIONS

This paper investigated user association and hybrid beamforming along with fronthaul compression design in C-RAN based mmWave cell-free systems. Max-min fairness problem was considered in this paper. We proposed to first jointly design the user association and analog beamforming to maximize the minimum accumulative beamforming gains among users. Then, given the effective baseband channel, the optimal digital beamformer and quantization noise covariance matrix are efficiently calculated based on fractional programming method. Simulation results demonstrated the effectiveness of the proposed algorithm, which can achieve comparable performance to the full-digital case.

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